Quadrupole Moments of Odd-Neutron Nuclei; Spin and Moments of 14-Year Cd^{113m}†

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The nuclear spin and hfs in the $(5s5p)\ ^3P_1$ state of 14-year Cd^{113m} have been measured by the optical double-resonance technique. The isotope was produced by the reaction $Cd^{112}(n,\gamma)Cd^{113m}$. After irradiation, the Cd^{113m} was separated from the Cd^{112} with an electromagnetic mass separator. The nuclear spin I, and the hfs separations are: $I=11/2$; $\nu(13/2-11/2)=4310.572(5)$ Mc/sec; $\nu(11/2-9/2)=3949.625(7)$ Mc/sec. The hfs coupling constants, corrected to second order for interaction with the $^3P_2$ and $^3P_0$ states, are: $A(113m)=-686.0425(8)$ Mc/sec; $B(113m)=+169.047(9)$ Mc/sec. If we neglect nuclear structure and quadrupole shielding effects, the nuclear moments are: $\mu(113m)=-1.08885(13)\mu_N$; $Q(113m)=-0.79(10)$ b. The ratio of the Cd^{113m} and Cd^{109} quadrupole moments is $Q(113m)/Q(109)=-1.02371(4)$; this result is independent of the shielding corrections. The spin and magnetic moment are consistent with a $(2d_{5/2})^6(1g_{7/2})^8(1h_{11/2})^1$ neutron assignment with some configuration mixing. We compare the observed quadrupole moments in cadmium and other spherical odd-neutron nuclei ($A<150$, $189<A<210$) with predictions of the shell model, including configuration mixing, and with the quasiparticle model of Kisslinger and Sorensen. In addition, we show that the semiempirical formula

$$Q=-[(2j+1-2N)/(2j+2)]Q_0(Z)$$

accurately predicts the quadrupole-moment ratios for the isotopes of several elements. We find that $Q_0(Z)$ exhibits strongly oscillatory dependence on the nuclear charge Z with minima near the magic proton numbers. We suggest that $Q_0(Z)$ may be interpreted in terms of the quadrupolar polarizability of the proton core.

I. QUADRUPOLE MOMENTS OF ODD-NEUTRON NUCLEI

At present, our understanding of nuclear theory is not sufficient to enable us to make many accurate statements about nuclear moments. Among the more general conclusions that we can make is that electric multipole operators are independent of the nuclear interaction currents and, therefore, that the electric moments are determined solely by the nuclear charge density. Hence, the observed quardupole moments provide critical tests for proposed nuclear wave functions. The magnetic moments, on the other hand, depend, in addition, on the assumed form of the nucleon interaction currents. Consequently, discrepancies between the observed and predicted magnetic moments must be attributed to uncertainties in both the wave functions and the Hamiltonian.[1]

The jj-coupling shell model of Mayer and Jensen,[2] which starts with a harmonic oscillator potential, a predominantly attractive spin-orbit potential, and a small l^2 correction, does not account at all for the observed quadrupole moments and only rather poorly for the observed magnetic dipole moments. In particular, this model predicts zero quadrupole moments for odd-neutron nuclei, and a quadrupole moment of

$$Q=[-(2j+1-2N)/(2j+2)]Q_0$$

for odd-proton nuclei, j being the total angular momentum of the last odd proton and N being the number of identical particles with the same quantum numbers as the last odd proton ($N\leq 2j+1$).[3] This group of N nucleons (N odd) which gives rise to the observed nuclear spin are referred to as the j-shell. Q_0 is roughly related to the nuclear radius and is often estimated as $Q_0=\frac{3}{5}r_0^2A^{2/3}$, where $r_0=1.2\times10^{-13}$ cm. These predictions are grossly inconsistent with the observation that the quadrupole moments for odd-neutron nuclei are of the same order of magnitude as those observed for odd-proton nuclei and that some nuclei exhibit very large moments.

Theories which make use of a deformed harmonic oscillator potential as the basis for an individual particle approach to nuclei, of which perhaps the most notable is the work of Mottelson and Nilsson,[4] have had considerable success in giving a semiphenomonological explanation of many nuclear properties in the region of heavy nuclei, particularly in providing the possibility of very large quadrupole moments. However, in the region below $A=150$ and around the "doubly magic" closed shell at Pb^{208}, this approach has not been particularly successful. The deformed nuclei have been adequately discussed in the literature,[5] and we will not consider them further. In the regions of nuclei $A<150$ and $189<A<210$, we will consider models which are

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[3] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953).
[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab Mat.-Phys. Medd. **29**, No. 16 (1955); B. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrift. **1**, No. 8 (1959).
[5] E. Marshalek, L. W. Person, and R. K. Sheline, Rev. Mod. Phys. **35**, 108 (1963).

TABLE I. Spins and nuclear moments in zinc, cadmium, and mercury. For magnetic dipole moments we list the Schmidt value, the theoretical value using the results of Arima and Horie [μ(Th)], and the experimental value. The values of μ(Th) were calculated with the pairing-energy parameter $C=30$ MeV, the value which gives the best agreement with nuclear binding energies. Unless otherwise specified, the experimental values are accurate to the number of decimal places given.

Isotope	Neutron configuration in last unfilled shell[a]	Spin	μ(Sch) in μ_N	μ(Th) in μ_N	μ(Exp) in μ_N
${}_{30}Zn_{35}^{65}$ [b]	$(2p_{3/2})^4(1f_{5/2})^3$	5/2	+1.37	+0.79	+0.77
${}_{30}Zn_{37}^{67}$ [c]	$(2p_{3/2})^4(1f_{5/2})^5$	5/2	+1.37	+1.02	+0.88
${}_{48}Cd_{59}^{107}$ [d]	$(2d_{5/2})^5(1g_{7/2})^4$	5/2	−1.91	−0.64	−0.62
${}_{48}Cd_{61}^{109}$ [e]	$(2d_{5/2})^5(1g_{7/2})^6$	5/2	−1.91	−0.76	−0.83
${}_{48}Cd_{63}^{111*}$ [f]	$(2d_{5/2})^5(1g_{7/2})^6(3s_{1/2})^2$	5/2	−1.91	−0.76	−0.78(3)
${}_{48}Cd_{63}^{111}$ [g]	$(2d_{5/2})^6(1g_{7/2})^6(3s_{1/2})^1$	1/2	−1.91	−0.98	−0.60
${}_{48}Cd_{65}^{113}$ [g]	$(2d_{5/2})^6(1g_{7/2})^8(3s_{1/2})^1$	1/2	−1.91	−1.11	−0.62
${}_{48}Cd_{65}^{113m}$ [h]	$(2d_{5/2})^6(1g_{7/2})^8(3s_{1/2})^0(1h_{11/2})^1$	11/2	−1.91	−1.37	−1.09
${}_{48}Cd_{67}^{115}$ [i]	$(2d_{5/2})^6(1g_{7/2})^8(3s_{1/2})^1(1h_{11/2})^2$	11/2	−1.91	−1.02	−0.65
${}_{48}Cd_{67}^{115m}$ [i]	$(2d_{5/2})^6(1g_{7/2})^8(3s_{1/2})^0(1h_{11/2})^3$	11/2	−1.91	−1.22	−1.04
${}_{80}Hg_{113}^{193}$ [j]	$(1i_{13/2})^{10}(2f_{5/2})^2(3p_{1/2})^1(3p_{3/2})^0$	1/2	+0.64	+0.64	+0.56(2)
${}_{80}Hg_{113}^{193m}$ [j]	$(1i_{13/2})^{11}(2f_{5/2})^2(3p_{1/2})^0(3p_{3/2})^0$	13/2	−1.91	−1.10	−1.06
${}_{80}Hg_{115}^{195}$ [k]	$(1i_{13/2})^{12}(2f_{5/2})^2(3p_{1/2})^1(3p_{3/2})^0$	1/2	+0.64	+0.64	+0.54
${}_{80}Hg_{115}^{195m}$ [k]	$(1i_{13/2})^{11}(2f_{5/2})^4(3p_{1/2})^0(3p_{3/2})^0$	13/2	−1.91	−0.99	−1.05
${}_{80}Hg_{117}^{197}$ [l]	$(1i_{13/2})^{12}(2f_{5/2})^4(3p_{1/2})^1(3p_{3/2})^0$	1/2	+0.64	+0.64	+0.52
${}_{80}Hg_{117}^{197m}$ [m]	$(1i_{13/2})^{13}(2f_{5/2})^4(3p_{1/2})^0(3p_{3/2})^0$	13/2	−1.91	−1.11	−1.03
${}_{80}Hg_{119}^{199}$ [n]	$(1i_{13/2})^{12}(2f_{5/2})^6(3p_{1/2})^1(3p_{3/2})^0$	1/2	+0.64	+0.64	+0.50
${}_{80}Hg_{119}^{199*}$ [o]	$(1i_{13/2})^{14}(2f_{5/2})^5(3p_{1/2})^0(3p_{3/2})^0$	5/2	+1.37	+1.06	+1.03(8)
${}_{80}Hg_{121}^{201}$ [n,p]	$(1i_{13/2})^{12}(2f_{5/2})^6(3p_{1/2})^0(3p_{3/2})^3$	3/2	−1.91	−0.48	−0.56
${}_{80}Hg_{123}^{203}$ [q]	$(1i_{13/2})^{14}(2f_{5/2})^5(3p_{1/2})^0(3p_{3/2})^4$	5/2	+1.37	+0.89	+0.83(2)

[a] In the mercury isotopes we omit the terms $(1h_{9/2})^{10}(2f_{7/2})^8$ which are common to all mercury isotopes.
[b] See Ref. 9.
[c] S. S. Dharmatti and H. F. Weaver, Jr., Phys. Rev. **85**, 927 (1952); A. Lurio, Phys. Rev. **126**, 1768 (1962).
[d] See Ref. 8.
[e] See Ref. 7.
[f] R. M. Steffen and W. Zobel, Phys. Rev. **103**, 126 (1956); H. J. Behrend and D. Budnick, Z. Physik **168**, 155 (1962).
[g] W. G. Proctor and F. C. Yu, Phys. Rev. **76**, 1728 (1949).
[h] This paper.
[i] See Ref. 10.
[j] These values are estimates based on preliminary results of W. W. Smith, W. T. Walter, and M. J. Staven, MIT Research Laboratory of Electronics, Quarterly Progress Report 70, 1963, pp. 33 and 39 (unpublished).
[k] W. W. Smith, MIT Research Laboratory of Electronics, Quarterly Progress Report 70, 1963, p. 33 (unpublished).
[l] W. T. Walter, Bull. Am. Phys. Soc. **7**, 295 (1962).
[m] H. R. Hirsch, J. Opt. Soc. Am. **51**, 1192 (1961).
[n] B. Cagnac, Ann. Phys. (Paris) **6**, 467 (1961).
[o] L. Grodzins, R. W. Bauer, and H. H. Wilson, Phys. Rev. **124**, 1897 (1961).
[p] M. N. McDermott and W. Lichten, Phys. Rev. **119**, 134 (1960).
[q] Estimated values based on results of O. Redi and H. H. Stroke, MIT Research Laboratory of Electronics, Quarterly Progress Report 71, 1963, p. 33 (unpublished).

based on a spherical potential with residual two-body forces.

It has been realized for a long time that if one assumes the existence of residual nucleon-nucleon interactions which are *not* taken into account by the average shell-model central potential, then substantial corrections to the shell model will occur. In fact, using a Hamiltonian of the form

$$\mathcal{H}=\mathcal{H}_{\text{shell}}+\sum_{i>j} V_{ij}$$

and proceeding via perturbation theory, Arima and Horie[6] have found very large corrections to the magnetic dipole moments, even though the ground-state wave function is not changed appreciably. Throughout this series of papers,[7–10] the Arima-Horie model has been used to interpret observed dipole moments in zinc and cadmium, and the success of the model in these two elements is striking. The details of the situation in cadmium and zinc are summarized in Table I, along with results on mercury, another even Z element which has been investigated in considerable detail. It is seen that the qualitative agreement is vastly improved over the simple Schmidt predictions, but, nevertheless, the quantitative agreement is, in some cases, no better than 50%. The average percentage discrepancy for the dipole moments given in Table I is about 20%, whereas the corresponding discrepancy with the Schmidt values is about 100%; i.e., on the average, the observed odd-neutron moments are about a factor of 2 smaller than the shell-model values. Despite this improvement, it is clear from an examination of Table I that the model of Arima and Horie, in general, cannot account for detailed trends in nuclear magnetic moments.

In the case of quadrupole moments, Arima and Horie have shown that for odd-*neutron* nuclei, the result of proton excitations (induced by interactions of the core with the odd-neutron shell) is to give rise to a quad-

[6] A. Arima and H. Horie, Progr. Theoret. Phys. (Kyoto) **12**, 623 (1954); H. Noya, A. Arima, and H. Horie, Progr. Theoret. Phys. (Kyoto) Suppl. **8**, 33 (1958).
[7] M. N. McDermott and R. Novick, Phys. Rev. **131**, 707 (1963).
[8] F. W. Byron, Jr., M. N. McDermott, and R. Novick, Phys. Rev. **132**, 1181 (1963).
[9] F. W. Byron, Jr., M. N. McDermott, R. Novick, B. Perry, and E. B. Saloman, Phys. Rev. **134**, A47 (1964).
[10] M. N. McDermott, R. Novick, B. Perry, and E. B. Saloman, Phys. Rev. **134**, B25 (1964).

rupole moment of the form

$$Q=-[(2j+1-2N)/(2j+2)]\tilde{Q}_0, \qquad (1)$$

where $\tilde{Q}_0$ is a sum of a large number of terms depending on the quantum numbers of the excited protons and on those of the last odd neutron. The factor $-(2j+1-2N)/(2j+2)$ is characteristic of shell-model quadrupole-moment calculations and arises from the necessity of taking into account the symmetry requirements on the identical nucleons in the j shell.

We may indeed conjecture[11] that such a factor will always enter into the form of Q for odd-neutrons in the shell model and that $\tilde{Q}_0$ will be an intrinsically positive quantity (a conjecture borne out in first-order calculations of Arima and Horie). $\tilde{Q}_0$ may be interpreted either in terms of a quadrupolar polarizability of the proton core or in terms of an "effective charge" of the odd neutrons. This latter interpretation has only limited validity, since it is found that substantially different values of the effective charge are required to explain the observed values for the different multipole moments in one and the same nucleus.[1]

Kisslinger and Sorensen[12] have considered a model of the spherical nuclei in which the residual two-body interactions are represented by two simple components, the pairing force suggested by work in superconductivity, and a long-ranged part represented by a quadrupole force. The main assumption of the work is that the low-lying states of spherical nuclei can be treated in terms of two basic excitations, quasiparticles and phonons. They consider only the particles outside of closed shells explicitly and derive various single-particle and collective properties. In the regions $60<A<150$ and $189<A<206$, they obtain satisfactory agreement with the observed energies. Unfortunately, the model in this form does not correctly predict the magnetic dipole and electric quadrupole moments. For this reason they find it necessary to include an additional short-range interaction of the form considered by Arima and Horie. The quadrupole moments obtained with this model arise from both quasiparticles and phonons. The effect of the configuration mixing is to enhance the quasiparticle contribution. In addition, it is found that the quasiparticle and phonon contributions to the quadrupole moment are of the same sign.

In Sec. II we present our observations on 14-year Cd^{113m}. In Sec. III we compare the predictions of the various models with all of the available experimental results, and we discuss the quasiempirical formula given in Eq. (1).

II. SPIN AND MOMENTS OF Cd^{113m}

A. Introduction

Here we report on an optical double-resonance study of the spin and nuclear moments of 14-year Cd^{113m}. Previously, we have reported on Cd^{109} (Ref. 7), Cd^{107} (Ref. 8), Zn^{65} (Ref. 9), Cd^{115} and Cd^{115m} (Ref. 10).

In Cd^{113m}, the dominant neutron configuration is assumed to be $(2d_{5/2})^6(1g_{7/2})^8(3s_{1/2})^0(1h_{11/2})^1$, in addition to a magic core of fifty neutrons. The single $h_{11/2}$ odd neutron makes this a case of special interest. Parity considerations indicate very little configuration mixing of the h orbital, whereas the $3s_{1/2}$ ground state is expected to be strongly mixed with $2d_{3/2}$, accounting for the anomalously small moments in the isotopes of cadmium with spin one-half. In view of the importance of this nucleus, we have measured the hfs intervals to high precision.

The Cd^{113m} may be produced by the reaction $Pd^{110}(\alpha,n)Cd^{113m}$. This method should yield an isotopically pure sample.[7] Palladium foils were bombarded by a beam of 20-MeV alpha particles in the Brookhaven cyclotron, and counting measurements indicated that sufficient Cd^{113m} atoms were produced for a double-resonance experiment. However, in spite of many precautions, surface contamination of the foils and trapping of the Cd^{113m} atoms in the palladium lattice prevented us from producing a suitable resonance cell. In view of these difficulties, it was decided to produce Cd^{113m} by neutron capture and subsequent isotopic enrichment with an electromagnetic mass separator.[10] A 10-mg sample of separated Cd^{112} was irradiated in the MTR reactor for six weeks. It was expected that this would yield 6×10^{15} atoms of Cd^{113m}. If we assume a 0.1% over-all efficiency in the Argonne mass separator[13] and a 3% recovery of the Cd^{113m} from the platinum mass separator target,[10] then we would expect to obtain 2×10^{11} atoms in the quartz resonance cell. The extremely low specific activity of the Cd^{113m} precluded a detailed study of the nuclear decay. However, the intensity of the double-resonance signals were of about the same magnitude as those observed in the Cd^{115m} work, showing that the estimate given above is essentially correct. The large thermal-neutron cross section for Cd^{113} insures that this isotope cannot survive the neutron irradiation. This fact and the mass separation assure the isotopic and isomeric identification of Cd^{113m}. In the seven-month period over which the measurements were made, no decay in the resonances attributed to Cd^{113m} was observed. This is consistent with the assigned 14-year half-life.[14]

B. Observations on Cd^{113m}

The apparatus used in the Zeeman measurements was essentially the same as that described in Ref. 9. Unpolarized light directed along the magnetic field was used to excite the atoms, producing only σ excitation ($\Delta M=\pm1$). The detector polarizer was oriented to accept only π light. Low-field Zeeman transitions were

[11] Due to one of us (F.W.B., Jr.).

[12] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).

[13] We are indebted to Dr. M. S. Freedman and Dr. O. Skillbreid of the Argonne National Laboratories for performing this separation.

[14] A. C. Wahl, J. Inorg. Nucl. Chem. 10, 1 (1959).

observed in the Cd^{113m} cell at fields corresponding to $g_F/g_J=4/143$, using a natural cadmium lamp to excite the atoms, and at fields corresponding to $g_F/g_J=2/11$ and 2/13, using a Cd^{113} lamp. This Zeeman spectrum can only be produced by a spin eleven-halves isotope, the former transitions corresponding to the central $F=11/2$ state and the latter two corresponding to $F=9/2$ and $F=13/2$, respectively. The large hfs in the eleven-halves isotope requires the use of the Cd^{113} lamp to illuminate the $F=13/2$ and 9/2 states, which lie outside the lamp profile of a natural cadmium lamp. These results establish the nuclear spin of Cd^{113m} to be $I=11/2$, in agreement with the earlier assignment based on the half-life of the nuclear decay.[14]

Transitions observed with circularly polarized light indicated that the Cd^{113m} hfs is inverted (see Ref. 7 and Ref. 8). Sample resonance curves are shown in Fig. 1. The inverted hfs implies that the nuclear magnetic moment is negative and that the last odd neutron is in a state with $I=l+\frac{1}{2}$. This agrees with the $h_{11/2}$ shell-model assignment.

The intermediate-field Zeeman dependence of transitions (see Fig. 2) within each of the three F states gave preliminary values of the hfs to be: $\nu(13/2-11/2)=4310(4)$ Mc/sec; $\nu(11/2-9/2)=3957(4)$ Mc/sec. This leads to the following preliminary values of the hyperfine interaction constants: $A(113m)=-686.6(4)$ Mc/sec; $B(113m)=+172(3)$ Mc/sec. Precise low-field direct determination of the hfs was undertaken in view of the theoretical interest in this nucleus (see Sec. II.A).

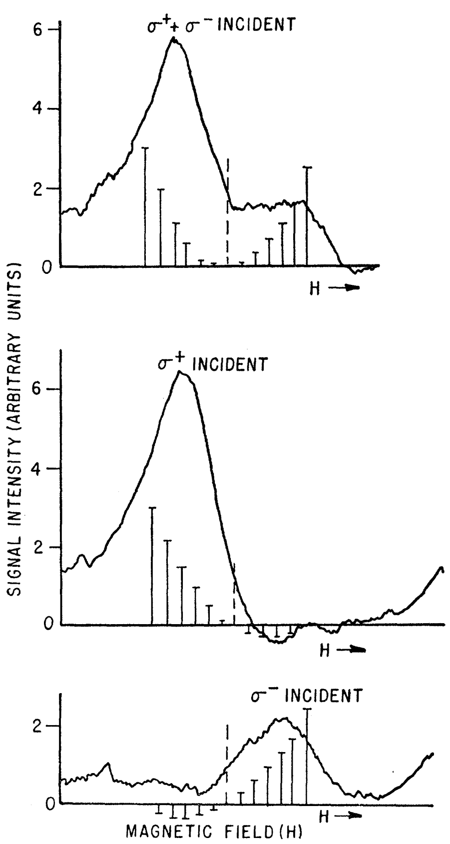

FIG. 1. Circular polarization observations in the $F=13/2$ state of Cd^{113m}. The vertical bars indicate the predicted position and intensity of the individual Zeeman resonances. These resonances indicate that the $^3P_1$ hfs of Cd^{113m} is inverted.

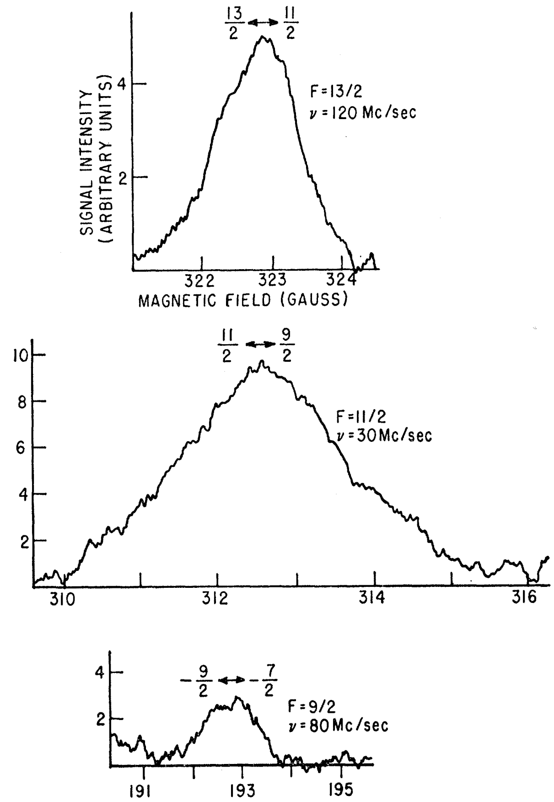

FIG. 2. Observed Zeeman resonances in Cd^{113m}.

In making direct hyperfine measurements in Cd^{113m}, two major difficulties had to be overcome. The large multiplicity of levels in a spin eleven-halves isotope greatly reduces the transition-signal strengths. Secondly, high-intensity rf fields are required to produce hfs transitions in an atom with large nuclear spin. The first difficulty was overcome by using a Cd^{114} lamp to produce excitations only into the $F=11/2$ central state. Following the method first developed by Kohler,[15] a Cd^{114} absorption cell was placed in the detection arm to reduce the effect of instrumental and atomic scattering without reducing the signal arising from transitions from the 11/2 to the 9/2 and 13/2 states. The light arising from these transitions falls outside of the absorption spectrum of a Cd^{114} bulb. This technique greatly enhances the signal-to-noise ratio for the direct hfs transitions.

To produce hfs transitions in a spin eleven-halves atom, a rf magnetic field of 0.64 G is required. To produce this field we used a 100-W air-cooled Litton L 3505

[15] R. H. Kohler, Phys. Rev. 121, 1104 (1961).

TABLE II. Constants used in the calculations of the second-order corrections to the Cd^{113m} $^3P_1$ hfs.

Constant	Value of the constant for cadmium
C_1	0.5402(9)
C_2	0.8416(6)
ζ	1.033
η	1.094
$5\theta(1-\delta)(1-\epsilon)$	6.42

magnetron tunable from 3.5 to 5.5 kMc/sec.[16] The power for the magnetron was obtained from a current regulared 3-kV 100-mA power supply which was square-wave modulated at 280 cps. Unfortunately, the magnetron frequency is sensitive to small variations in the current, and unavoidable fluctuations in the power supply produced some frequency modulation. The high-frequency transitions were identified by their Zeeman effect. Transitions were observed in pairs $[F,M]\leftrightarrow[F',(M\pm1)]$ and $[F,-M]\leftrightarrow[F',-(M\pm1)]$. The members of these pairs have equal and opposite linear Zeeman displacements.

Precise observations were made on the following pairs of transitions: $(13/2,\pm1/2)\leftrightarrow(11/2,\pm3/2)$ and $(11/2,\pm3/2)\leftrightarrow(9/2,\pm1/2)$. Each transition was observed at three different values of the magnetic field. These measurements lead to the following values for the zero-field intervals:

$$\nu(13/2-11/2)=4310.572(5)\ \text{Mc/sec},$$
$$\nu(11/2-9/2)=3949.625(7)\ \text{Mc/sec}.$$

Under the conditions required for observing the (13/2–11/2) transitions, the incidental frequency modulation of the magnetron broadened its spectrum to about 100 kc/sec, but the central frequency was very stable. In the case of the (11/2–9/2) transitions, the FM broadening was only about 10 kc/sec, but the mean frequency drifted over a somewhat larger interval. In all cases, the broadening of the magnetron spectrum was appreciably less than the atomic linewidth. At least ten frequency determinations were made for each resonance in order to reduce the effect of the magnetron frequency instabilities. The uncertainties in the final results are based on the internal consistency of the data, and the values given are twice the standard deviation.

Off-diagonal hfs interactions between the $^3P_1$, $^3P_2$, and $^3P_0$ states produce a modification of the apparent interaction constants. The method of Lurio, Mandel, and Novick[17] was used to correct for these interactions in calculating the constants from the hfs intervals. The constants used in evaluating the corrections are given in Table II and are defined in Ref. 17. C_1 and C_2 were calculated from $\tau(^1P_1)$, determined by Lurio and Novick,[18] and $\tau(^3P_1)$, determined by Byron, McDermott, and Novick.[19] In Table III are given the individual electron coupling constants for Cd^{113m}. The corrected hfs coupling constants are

$$A(113m)=-686.0425(8)\ \text{Mc/sec},$$
$$B(113m)=+169.047(9)\ \text{Mc/sec}.$$

C. Nuclear Moments of Cd^{113m}

The magnetic moment can be determined from $A(113m)$, if we neglect the effects of the finite nuclear size. That is, we assume that the ratio of the nuclear g factors of Cd^{113m} and Cd^{111} is equal to the ratio of $A(113m)$ to $A(111)$. For $A(111)=-4123.81(1)$ Mc/sec[20] and $\mu(111)=-0.59501(8)\mu_N$,[21] we find a value for the Cd^{113m} moment of $\mu(113m)=1.08885(13)\mu_N$. The uncertainty results entirely from the uncertainty in $\mu(111)$. No allowance has been made for the effects of the finite nuclear size.

The ratio of the quadrupole moments of Cd^{113m} and Cd^{109} is just the ratio of their quadrupole interaction constants. Thus, $Q(113m)/Q(109)=-1.02371(4)$, using $B(109)=165.143(5)$ Mc/sec from level-crossing measurements.[22] The details of the connection between the quadrupole interaction constant $B(109)$ in the $^3P_1$ state of cadmium and the quadrupole moment $Q(109)=+0.78(10)$ b are discussed in Ref. 7. From the ratio of the quadrupole moments stated, we find $Q(113m)=-0.79(10)$ b.

In Sec. I, we discussed the calculations of magnetic moments with the configuration mixing model of Noya, Arima, and Horie. In these calculations, we have used the value of the pairing energy parameter which best fits the binding energy data, $C=30$ MeV. A comparison of the observed and predicted moments is given in Table I for the zinc, cadmium, and mercury isotopes. As in the case of Zn^{65}, Cd^{115}, and Cd^{115m}, this model does

TABLE III. Individual electron hyperfine coupling constants for Cd^{113m}.

Constant	Value of the constant in Mc/sec
a_s	−2058.9(25)
$a_{1/2}$	−281.0(5)
$a_{3/2}$	−42.3(8)
$b_{3/2}$	−303.2(9)

[16] The carbon blackening on the interior of tuned cavities, necessary to reduce instrumentally scattered light, reduced the Q of the cavities and made it necessary to have an rf power of greater than 10 W.

[17] A. Lurio, M. Mandel, and R. Novick, Phys. Rev. **126**, 1758 (1962).

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[19] F. W. Byron, Jr., M. N. McDermott, and R. Novick, Phys. Rev. **134**, A615 (1964).

[20] R. F. Lacey (private communication). A preliminary result appeared in MIT Research Laboratory of Electronics Quarterly Progress Report, 1959, p. 49 (unpublished).

[21] W. G. Proctor and F. C. Yu, Phys. Rev. **79**, 35 (1950).

[22] P. Thaddeus and M. N. McDermott, Phys. Rev. **132**, 1186 (1963).

TABLE IV. The spins and quadrupole moments of odd-neutron spherical nuclei. The predictions of the configuration mixing (CM) model and the quasiparticle (QP) model are given where calculations have been carried out. Also given are the shell-model assignment for each nucleus and the value of $\tilde{Q}_0$ obtained from Eq. (1). The quadrupole moments are given in barns.

Isotope	I	Q[a]	Q(CM)[b]	Q(QP)[c]	Neutron configuration in addition to filled shells	$\tilde{Q}_0/A^{2/3}$
Be^9	3/2	+0.03			$(1p_{3/2})^3$	0.017
C^{11}	3/2	+0.03[d]			$(1p_{3/2})^3$	0.015
O^{17}	5/2	−0.03	−0.027		$(1d_{5/2})^1$	0.008
Mg^{25}	5/2	+0.22			$(1d_{5/2})^5$	0.045
S^{33}	3/2	−0.064	−0.05		$(1d_{3/2})^1$	0.016
S^{35}	3/2	+0.054	+0.05		$(1d_{3/2})^3$	0.013
Cr^{53}	3/2	−0.03			$(2p_{3/2})^1$	0.005
Fe^{57m}	3/2	+0.24(4)[e]			$(2p_{3/2})^3$	0.04
Zn^{65}	5/2	−0.027	0.0	+0.59	$(1f_{5/2})^3$	
Zn^{67}	5/2	+0.16	+0.14	+0.66	$(1f_{5/2})^5$	0.017
Ge^{73}	9/2	−0.2	−0.16	−0.99	$(1g_{9/2})^1$	0.016
Kr^{83}	9/2	+0.27	+0.17	+0.89	$(1g_{9/2})^7(2p_{1/2})^2$	0.036
Kr^{85}	9/2	+0.45	+0.35	+0.89	$(1g_{9/2})^9(2p_{1/2})^2$	0.032
Sr^{87}	9/2	+0.36		+0.53	$(1g_{9/2})^9(2p_{1/2})^2$	0.025
Cd^{107}	5/2	+0.79	+0.10[f]	+0.79	$(2d_{5/2})^5(1g_{7/2})^4$	0.061
Cd^{109}	5/2	+0.78		+0.75	$(2d_{5/2})^5(1g_{7/2})^6$	0.060
Cd^{111*}	5/2	+0.9(3)		+0.62	$(2d_{5/2})^5(1g_{7/2})^6(3s_{1/2})^2$	0.068
Cd^{113m}	11/2	−0.81			$(1h_{11/2})^1$	0.045
Cd^{115m}	11/2	−0.61			$(1h_{11/2})^3$	0.056
Sn^{119}	3/2	−0.08			$(2d_{3/2})^1(1h_{11/2})^4$	0.008
Te^{125}	3/2	−0.20			$(2d_{3/2})^1(1h_{11/2})^8$	0.020
Xe^{129m}	3/2	(−)0.42(5)[g]		+0.19	$(2d_{3/2})^1(1h_{11/2})^8$	0.041
Xe^{131}	3/2	−0.12	−0.14	+0.57	$(2d_{3/2})^1(1h_{11/2})^{10}$	0.012
Ba^{135}	3/2	+0.13[h]		+0.47	$(2d_{3/2})^3(1h_{11/2})^{10}$	0.012
Ba^{137}	3/2	+0.20[h]		+0.26	$(2d_{3/2})^3$	0.019
Nd^{143}	7/2	−0.48		−1.06	$(2f_{7/2})^1$	0.026
Nd^{145}	7/2	−0.26		−0.56	$(2f_{7/2})^3$	0.042
Sm^{147}	7/2	−0.21		−1.19	$(2f_{7/2})^3$	0.034
Sm^{149}	7/2	+0.06		−1.29	$(2f_{7/2})^5$	0.010
Os^{189}	3/2	+0.8	+0.10	−0.36	$(3p_{3/2})^3(1i_{13/2})^{10}$	0.06
Hg^{193m}	13/2	+1.3(4)[i]			$(2f_{5/2})^2(1i_{13/2})^{11}$	0.07
Hg^{195m}	13/2	+1.4(6)			$(2f_{5/2})^4(1i_{13/2})^{11}$	0.07
Hg^{197m}	13/2	+1.6(1)		+1.55	$(2f_{5/2})^4(1i_{13/2})^{13}$	0.059
Hg^{201}	3/2	+0.50	+0.07	+0.71	$(3p_{3/2})^3(1i_{13/2})^{12}$	0.036
Hg^{203}	5/2	+0.5(8)[j]			$(2f_{5/2})^5$	0.03
Po^{205}	5/2	+0.17			$(2f_{5/2})^5$	0.009
Po^{207}	5/2	+0.28			$(2f_{5/2})^5$	0.014

[a] Except where otherwise stated we have used the following compilation of quadrupole moments: I. Lindgren, *Perturbed Angular Correlations*, Appendix I, Table of Nuclear Spins and Moments (North-Holland Publishing Company, Amsterdam, 1964).
[b] See Ref. 6.
[c] See Ref. 12.
[d] R. A. Haberstroh, thesis, Princeton University, New Jersey, Palmer Physical Laboratory, 1963 (unpublished).
[e] R. Ingalls, Phys. Rev. **128**, 1155 (1962); R. M. Sternheimer, Phys. Rev. **130**, 1423 (1963); H. Eicher, Z. Physik **171**, 582 (1963).
[f] See Ref. 8.
[g] G. J. Perlow, Bull. Am. Phys. Soc. **9**, 11 (1964).
[h] D. A. Jackson and D. H. Tuan, Phys. Rev. Letters **11**, 209 (1963); G. Zu Putlitz, Ann. Phys. **11**, 248 (1963); Allen Lurio, Phys. Rev. **136**, A376 (1964).
[i] See Ref. j, Table I.
[j] O. Redi and H. H. Stroke, Bull. Am. Phys. Soc. **9**, 10 (1964).

not entirely account for the reduction of the magnetic moment from the Schmidt limit. We have investigated other values for the coupling parameter C and find that the choice $C=40$ MeV gives slightly more satisfactory agreement between the observed and calculated moments in the zinc isotopes, and that $C=60$ MeV gives excellent agreement in the spin one-half and spin eleven-halves cadmium isotopes. However, since these values are not consistent with the observed pairing energies, we will not consider them further.

III. DISCUSSION

A. Survey of Odd-Neutron Quadrupole Moments and Neutron Assignments

In Sec. I, we discussed the theories which have been used by Noya, Arima, and Horie and by Kisslinger and Sorensen to compute the nuclear moments of spherical nuclei. We noted that the comparison of predicted electric quadrupole moments with experimental values provides a critical test of proposed nuclear wave functions. In Table IV we list all of the quadrupole moments which have been measured in odd-neutron nuclei for $A<150$ and $189<A<210$, along with the values predicted on the basis of the two models. In addition, we give the shell-model assignment for each nucleus and the value of $\tilde{Q}_0$ obtained from Eq. (1).

The precision of the quadrupole moments is limited by the well-known uncertainties in our knowledge of the electric-field gradient at the nucleus. Where quadrupole moments of several isotopes of an element have been measured by the same technique, the precision of the ratio of the quadrupole moments is limited only by the experimental uncertainties. For example, in the work on cadmium we have determined the $Q(\mathrm{Cd}^{113m})/Q(\mathrm{Cd}^{109})$ ratio to four parts in 10^5 (see Sec. II.C). However, various estimates[7] of the field gradient differ by about 10%. In addition, we expect that the quadrupole shielding corrections, which have not been included in our calculations, may be of comparable magnitude. Thus the absolute moments may be in error by as much as 20%.

In order to test the validity of the conjecture summarized by Eq. (1), it is clearly necessary that we obtain level assignments which are not based on the observed quadrupole moments. This problem is particularly acute in the heavy elements such as mercury where the levels are so numerous and so nearly degenerate that we may make several possible assignments which are consistent with the observed spin, parity, and energy. In such cases we have chosen the assignment that gives the best agreement with the magnetic moment. Below, we comment on the entries in Table IV.

1. Light Nuclei

In this region, we have used the configurations of Noya, Arima, and Horie.[6] These assignments may be chosen unambiguously since the number of single-particle levels is small, and they are well separated in energy. Nuclei in which the ground state has seniority greater than one have not been included, since the quadrupole moment for the coupled nucleons in this case bears no simple relation to the reduced moment of an individual nucleon. There is increasing evidence that Ne^{21} should be treated by the Nillson model and that collective effects are important in the selenium isotopes.[23,24]

The only nuclei in this region whose quadrupole moments are in serious doubt are Fe^{57*} and Cr^{53}. In the case of Fe^{57}, the Mössbauer measurements at Fe^{++} and Fe^{3+} sites deviate by 40%, and in the case of Cr^{53} only the order of magnitude of the absolute value is known.

2. Cadmium through Samarium; Osmium through Polonium

These two regions center about the filled shells at $Z=50$, $N=82$ and $Z=82$, $N=126$, respectively. Those odd nuclei away from closed shells exhibit large quadrupole moments, and their even-even neighbors exhibit vibrational character. These observations imply the onset of collective effects. The neutron shells lie very close together in energy, so that particular attention must be paid to the competition between the single-particle energies and the pairing energies which tend to couple pairs of particles into shells with large angular momenta. For those cases where there is any ambiguity, we have used the configuration which gives the best prediction, on the basis of the configuration mixing model, of the observed magnetic dipole moment. Table I shows these predictions for cadmium and mercury. For other elements, the reader should consult Ref. 6. Except in the case of Cd^{111*} and Xe^{129*}, the quadrupole moments of all the isotopes of a given element were determined by the same technique. The Cd^{111*} quadrupole moment was determined with a γ-γ correlation technique in a solid sample rather than by the optical double-resonance method which was employed for the other cadmium isotopes. The Xe^{129*} moment was determined by Mössbauer technique rather than by the spectroscopic techniques used for Xe^{131}. Unfortunately, the sign of the Xe^{129*} moment was not determined in the Mössbauer work.

B. Comparison of Observed Quadrupole Moments with Theory

The Noya, Arima, and Horie model is outstandingly successful in predicting the quadrupole moments of the light nuclei; we note, in particular, sulfur, zinc, and krypton. Outside of this region they have calculated few moments for the odd-neutron spherical nuclei. The value for Xe^{131} is the only one of these predictions which is in good agreement with the observed quadrupole moment. This nucleus lies close to the major shell closure at $Z=50$, $N=82$, and it is expected that other moments in this region could be calculated successfully on the basis of their model. The two values they calculate in the heavy nuclei near the major closed shells at $Z=82$, $N=126$ are in poor agreement with experiment.

Kisslinger and Sorensen have not considered nuclei lighter than $A=60$, and in the region $60<A<85$ their calculated moments are at least a factor of 2 greater than the measured values. For the four cases Sr^{87}, Cd^{107}, Cd^{109}, and Cd^{111*}, the quasiparticle predictions agree with the measured values within about 40%. Except for sporadic cases such as Ba^{137} and the two isotopes of mercury, the remaining predictions are not satisfactory. As we indicated in Sec. I, the basic pairing-plus-quadrupole-force model was modified, and a number of *ad hoc* assumptions were introduced to yield even this rather limited agreement.

The quadrupole moments obtained by Noya, Arima, and Horie are based on a first-order calculation of the deformation of the proton core by the neutrons in the j shell via residual two-body interactions. In the case of heavy nuclei with many close-lying levels, we would expect that a higher-order treatment would be required, and, in fact, we have noted that the Noya, Arima, and Horie results do not agree with experiment in such regions. In Sec. I, we have conjectured that the factor $-(2j+1-2N)/(2j+2)$ which appears in their theory will also appear in a more complete (i.e., higher order) treatment of the quadrupole moments. In the spirit of

[23] B. E. Chi and J. P. Davidson, Phys. Rev. **131**, 366 (1963); R. M. Dreizler, Phys. Rev. **132**, 1166 (1963); J. R. Roesser and J. P. Davidson, Bull. Am. Phys. Soc. **9**, 416 (1964).
[24] W. R. Wiseman and R. M. Williamson, Nucl. Phys. **23**, 536 (1961).

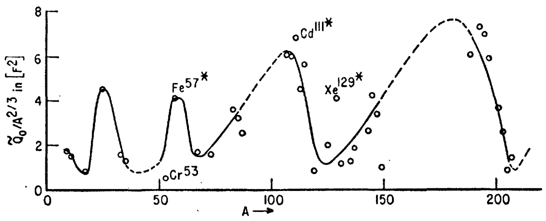

FIG. 3. A plot of $\tilde{Q}_0/A^{2/3}$ versus A, the nuclear mass number. $\tilde{Q}_0$ is defined in the text. The curve has been drawn to indicate the regularities of the moments. Experimental values which are in particular doubt are indicated by their mass numbers and are discussed in the text. The quadrupole moments are given in units of F^2 (10^{-26} cm^2).

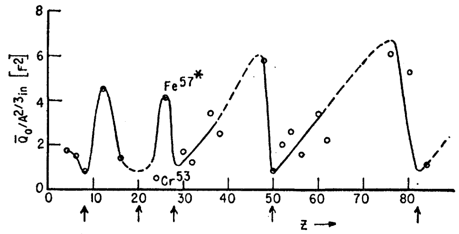

FIG. 4. A plot of $\bar{Q}_0/A^{2/3}$ versus Z, the nuclear charge. $\bar{Q}_0$ is the average value of $\tilde{Q}_0$ for each element. The two cases with large experimental uncertainty, as discussed in the text, are indicated by their mass numbers. The smooth curve has been drawn to indicate the regularities in the averaged moments. The arrows indicate the major shell closures. The quadrupole moments are given in units of F^2 (10^{-26} cm^2).

this conjecture, we might expect that the factor $\tilde{Q}_0$ in Eq. (1) will depend strongly on the number of protons in the core and somewhat more weakly on the neutron number.

Using the shell-model assignments listed in Table IV and discussed in Sec. III.A, we obtained values of $\tilde{Q}_0$ for all of the odd-neutron spherical nuclei.[25] These values are listed in column 7 of Table IV and are plotted against A in Fig. 3. The most striking conclusion that we obtain is the fact that all of the values of $\tilde{Q}_0$ so obtained are positive. Moreover, it is clear that $\tilde{Q}_0$ depends not only on the proton number but also on the neutron number. The dependence on the neutron number is greatest in the heavy nuclei and is particularly evident in the mercury isotopes.

In Fig. 4, we have plotted the average value of $\tilde{Q}_0$, $\bar{Q}_0$ for each element. It is seen that $\bar{Q}_0$ appears as an intrinsically positive quantity as indicated above, and that the curve has an oscillatory character of the type usually associated with a Townes-Low-Foley plot.[26] Since neutron and proton major closed shells generally occur together in the Periodic Table, it is difficult to be certain whether or not the minima occur at the closing of the proton shell, as our plot against Z would seem to suggest or at the closing of the neutron shells.

We should emphasize at this point the difference between Fig. 4 and the Townes-Low-Foley plot; it is, namely, that we plot against the number of protons and not against the number of odd neutrons. Firstly, we expect that after taking out the factor $-(2j+1-2N)/(2j+2)$ we will find the $\tilde{Q}_0$ for the various isotopes of a given element to be roughly equal, and for that reason, a plot against the neutron number is not particularly illuminating. Secondly, since we are trying to look at the problem from the point of view of the usefulness of perturbation theory, the protons are particularly relevant, especially in the case of odd-neutron even-proton nuclei, where the quadrupole moment is to be imagined as an entirely induced effect, arising from the interaction of the neutrons in the last j shell with the core proton shells.

If one makes the same plot for odd-proton nuclei, one again finds Q_0 to be an intrinsically positive quantity, except in the case of V^{51}.[27] Because of the possibility of excitations into and out of the j shell itself, which is a proton shell in this case, we would not expect the factor $-(2j+1-2N)/(2j+2)$ to give such a good representation of the experimental facts. Similarities between the odd-neutron even-proton case and the odd-proton even-neutron case are apparent, but any comparison between the two is unwarranted because of the necessary crudeness of the analysis.

[25] The j-shell in Zn^{65}, as indicated in Table IV, is half filled, so that the factor $(2j+1-2N)$ is zero. Since the observed Zn^{65} quadrupole moment is finite, the corresponding value $\tilde{Q}_0$ will be infinite. However, since the measured moment is smaller than any other measured in odd-neutron nuclei, we believe that this is essentially consistent with the prediction of $Q=0$.

[26] C. H. Townes, H. M. Foley, and W. Low, Phys. Rev. **76**, 1415 (1949).

[27] It has been suggested that the configuration for V^{51} should be $(1d_{3/2})^{-2}(1f_{7/2})^5$ rather than $(1f_{7/2})^3$; [H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955)]. It is also true that the configuration $(1d_{3/2})^{-2}(1f_{7/2})^3$ gives a much better magnetic moment value for Sc^{45} than does $(1f_{7/2})^1$. However, given the "magic" character of nuclei with 20 protons, these possibilities seem unlikely.